

N74-14609

## SUBSONIC FLUTTER ANALYSIS ADDITION TO NASTRAN

Robert V. Doggett, Jr.  
NASA Langley Research Center  
Hampton, Virginia

and

Robert L. Harder  
The MacNeal-Schwendler Corporation  
Los Angeles, California

### SUMMARY AND ABSTRACT

A subsonic flutter analysis capability has been developed for NASTRAN, and a developmental version of the program has been installed on the CDC 6000 series digital computers at the Langley Research Center. The flutter analysis is of the modal type, uses doublet lattice unsteady aerodynamic forces, and solves the flutter equations by using the k-method. Surface and one-dimensional spline functions are used to transform from the aerodynamic degrees of freedom to the structural degrees of freedom. Some preliminary applications of the method to a beamlike wing, a platelike wing, and a platelike wing with a folded tip are compared with existing experimental and analytical results.

### INTRODUCTION

The available standard level of the NASA structural analysis computer program (NASTRAN) can be used to solve flutter problems by using the "direct input matrix" feature of the program to add the required unsteady aerodynamic force matrices to the appropriate structural matrices and solve the resulting eigenvalue problem. This procedure is inefficient and is not routinely used by aeroelasticians. However, since its first public release in 1970, NASTRAN has proven to be a very useful tool to many persons interested in flutter, but this use has been limited to using the program to calculate the structural modes and frequencies that are required as input to separate special-purpose flutter analysis computer programs. This use of NASTRAN by aeroelasticians has created some interest in incorporating a flutter analysis capability in NASTRAN. At the first NASTRAN Users' Experiences Colloquium (ref. 1) a paper (ref. 2) was presented that described the results of a design study for a complete NASTRAN aeroelastic analysis capability. By using this design study as a guideline, the NASA has sponsored the development of a subsonic flutter analysis addition to NASTRAN.

The purpose of this paper is to describe this new flutter analysis capability and present some results from preliminary applications of the program.

The technique developed is of the modal type, uses doublet lattice unsteady aerodynamic forces, uses one-dimensional and surface spline functions to transform from aerodynamic degrees of freedom to structural degrees of freedom, and solves the flutter equations by using the k-method. The program is in what might be termed a developmental form, has only been installed on the CDC 6000 series digital computers at the Langley Research Center, and is not available for general release to the public. Results from preliminary applications of the program to a beamlike wing, a platelike wing, and a platelike wing with a folded tip are compared with existing analytical and experimental results.

#### OBJECTIVES AND GUIDELINES

The basic steps required in a flutter analysis are shown in the block diagram presented in figure 1. A characterization of each step is shown on the right in the figure. The overall objective of the NASTRAN subsonic flutter analysis was to provide a fully automated means for proceeding through these steps in an efficient manner to determine the flutter characteristics of complex structural and aerodynamic configurations. The development was constrained to the use of existing, proven state-of-the-art techniques. Therefore, the major effort was to assemble the selected procedures into the NASTRAN environment. One of the most significant guidelines was that there would be no constraints imposed on the structural idealization by aerodynamic considerations, and that the aerodynamic idealization would be made totally independent of structural modeling considerations. That is, the structure can be represented by an optimum selection and arrangement of structural elements and degrees of freedom, and the aerodynamic characteristics can be determined by an optimum selection of aerodynamic degrees of freedom. This guideline dictated providing a very general capability for the structural-aerodynamic interface which is required to transform the aerodynamic degrees of freedom to the structural degrees of freedom. Additional guidelines were that the technique should be easy to use and that the input data requirements associated with the unsteady aerodynamics and flutter solution and the format of the output results be in a form not totally unfamiliar to aeroelasticians. Another guideline that should be mentioned is that, where practical, the new procedures required for flutter analysis would be made as general as possible so that the basic capability can be easily expanded, if so desired at a later date, to accommodate additional aerodynamic theories, flutter solution procedures, and so forth. Naturally, it was required that the flutter analysis be compatible with the existing NASTRAN general structural capability and contain such existing features as the restart capability. Further, it was required that the flutter analysis be incorporated into a standard level version (level 15.1 was chosen) so that the NASTRAN program which contains the flutter analysis will also have all the other basic capabilities.

#### METHOD IMPLEMENTED

A modal flutter analysis method has been implemented in NASTRAN. The set of linear equations of motion that must be solved to determine the flutter condition may be expressed in matrix notation in the following form:

$$\left[ \left[ \frac{k^2}{b^2} [M] + \frac{\rho}{2} [Q(M_0, k)] \right] \lambda^2 + [K] \right] \{ u_h \} = 0 \quad (1)$$

here

- M = generalized structural mass
- K = generalized structural stiffness
- Q = generalized unsteady aerodynamic force (function of  $M_0$  and  $k$ )
- b = reference length
- $M_0$  = Mach number
- k = reduced frequency,  $b\omega/V$
- V = velocity
- $u_h$  = generalized modal coordinate
- $\rho$  = fluid density
- $\lambda$  = complex eigenvalue

NASTRAN already contains the capability of generating the generalized mass and stiffness matrices required by equation (1) but does not contain any internal aerodynamic force capability. So one of the major tasks was to add the required unsteady aerodynamics. Since an important objective was to be able to analyze the most general aerodynamic configurations possible, the doublet lattice unsteady aerodynamics method was selected for inclusion since this method is applicable to a broad range of configurations. The flutter solution method implemented was the k-method which is the one most commonly used in flutter analysis. A modal formulation was chosen for two reasons. The first reason is that this is standard practice; the second reason is that the order of the final matrix equations that must be solved is relatively small. The aerodynamic-structural interface is accomplished by the use of one-dimensional and surface spline functions.

#### The k-Method of Solution

The k-method of flutter solution requires the repeated solution of equation (1). The aerodynamic forces are functions of the three parameters, density, Mach number, and reduced frequency. To solve equation (1) values of two of the parameters, usually density and Mach number, are held constant, and the eigenvalue equation is solved repeatedly for different values of reduced frequency. The way equation (1) is developed, the damping, velocity, and frequency of the system can be determined from the eigenvalues by using the relationships

$$g = 2 \lambda_{\text{REAL}} / \lambda_{\text{IMAG}}$$

$$f = k \lambda_{\text{IMAG}} / 2\pi b$$

$$V = \lambda_{\text{IMAG}}$$

Since the flutter point is on the boundary between stable (damped) and unstable (divergent) sinusoidal oscillations, the flutter condition occurs for the particular combination of parameters that causes the damping to equal zero ( $g = 0$ ). The flutter velocity is usually determined by graphically plotting the damping versus velocity ( $g - V$  plots) obtained for each solution of the eigenvalue problem. A number of loci, equal to the order of the problem, will be obtained. The curve which crosses the  $g = 0$  axis at the lowest value of velocity determines the critical flutter condition. The k-method implemented in NASTRAN includes the generation of both damping and frequency versus velocity plots ( $f - V$  plots). Also, the capability is provided for selecting any one of the three aerodynamic parameters as the one to be varied.

### Unsteady Aerodynamic Theory

The unsteady aerodynamic theory implemented in the NASTRAN flutter analysis is the subsonic doublet lattice method (ref. 3). Of the available proven theories, this technique is probably the most general in that it can be applied to multiple nonplanar mutually interfering lifting surfaces and can be used to calculate body-lifting surface interference effects. The doublet lattice method adapted for NASTRAN use is similar to that described in references 4 and 5. The program described in these references includes slender-body aerodynamics to calculate body, or fuselage, forces but this feature has not been included in NASTRAN although the work required to implement body forces has been determined.

The doublet lattice method requires that the aerodynamic surfaces be subdivided into a grid of trapezoidal boxes. An example box arrangement is illustrated in figure 2. The analyst is required to specify the box arrangement subject to certain geometric constraints. For example, two of these constraints are that the boxes must be arranged in streamwise columns parallel to the free stream and that surface discontinuities such as fold lines must lie on box boundaries. The geometric constraints on the box arrangement are not severe and provide sufficient latitude to model adequately very general configurations. For the unsteady flow case, a spanwise line of acceleration potential doublets is placed at the one-quarter-chord station of each box. The doublets are related to pressure and hence to the force on each box. An aerodynamic influence coefficient matrix is generated which relates the force on the boxes to the downwash on the boxes. The force acts at the one-quarter-chord point and the downwash point is the three-quarter-chord point. Both of these points are at the box midspan station. Typical force and downwash points are shown in figure 2. The downwash is a function of the streamwise slope and the vertical displacement normal to the boxes. Each box may be thought of in the context of being a finite element with the degrees of freedom (deflection at one-quarter-chord point, and deflection and slope at three-quarter-chord point) defined at two different points within each box. In the NASTRAN flutter development, it was decided that it would be desirable to have only one aerodynamic grid point for each box. The point selected was the center of each box. A transformation is used to convert the force and downwash at the one-quarter and three-quarter-chord points of each box to corresponding forces and downwashes at the centers of each box. Therefore, there is one aerodynamic grid point for each box.

## Structural-Aerodynamic Interface (Geometry Interpolation)

One of the most significant features of the NASTRAN flutter analysis is the geometry interpolation capability that provides for the interconnection of the aerodynamic and structural models of the system. Since a very general capability is provided for the structural-aerodynamic interface, the structural model can be that best suited from structural considerations alone, and the choice of aerodynamic model is dictated by aerodynamic considerations alone. The geometry interpolation provides a transformation from the aerodynamic degrees of freedom to the structural degrees of freedom. This transformation is accomplished by the use of one-dimensional and surface spline functions. (See refs. 6 and 7.) The traditional one-dimensional spline has been generalized to include torsional rotations in addition to bending deformations. Since these functions are based on the small deflection equations of infinite beams and plates, respectively, they are very good for the interpolation of the deformations of general structural systems. If the structure is expected to behave like a beam as would be the case for a high-aspect-ratio jet transport wing, the one-dimensional spline would be used; if the structure is expected to behave like a plate, say a low-aspect-ratio wing, the surface spline would be the appropriate choice. The use of combinations of the two splines is permissible and would be applied, for example, to a complete aircraft where the fuselage had the character of a beam and the wing was expected to exhibit platelike behavior.

## Aerodynamic Force Interpolation

The k-method type flutter solution requires the solution of the flutter eigenvalue problem many times so that a relatively closely spaced sequence of points can be determined to make the  $g - V$  plots since the behavior of the loci of roots on the plot can often be quite complex and lead to misinterpretation of the results. Since one of the most expensive parts of a flutter analysis is the determination of the unsteady aerodynamic forces, it is desirable to actually calculate the aerodynamic forces for a minimum number of values of the independent aerodynamic parameter, Mach number, or reduced frequency. Fortunately, experience has shown that although the behavior of the solutions of the flutter equations as displayed on a  $g - V$  diagram may be complex, the variation of the aerodynamic forces with reduced frequency or Mach number is generally smooth and well behaved. Consequently, it has become more or less standard practice in aeroelasticity to evaluate the aerodynamic forces at a relatively small number of values of the independent variable and interpolate to determine the forces at additional values of the independent parameter. This interpolation is relatively inexpensive when compared to the cost of actually calculating the aerodynamic forces and results in the loss of very little accuracy. Aerodynamic force interpolation has been included in the NASTRAN flutter analysis. Both one-dimensional and surface splines are used. If the flutter calculations are limited to a constant Mach number, the linear spline is used to interpolate over a range of reduced frequencies. If a set of aerodynamic forces have been determined at two or more Mach numbers, the surface spline is used to interpolate to intervening Mach numbers. Experience with the one-dimensional spline has shown that it is very good for aerodynamic

interpolation. However, there are some indications that the accuracy of the surface spline technique, although it is satisfactory, is not as good as the linear spline. This is probably caused by the fact that the character of the three-dimensional behavior of the aerodynamic forces is not plate-like.

#### FLUTTER ANALYSIS RIGID FORMAT

The assembly of the components of the flutter analysis into a NASTRAN rigid format (labeled Rigid Format 45) required the use of many existing functional modules, the modification to a few existing modules, and the development of six completely new modules. An annotated block diagram of the new rigid format is presented in figure 3. The structural analysis section is essentially identical to existing Rigid Format 10 (Modal Complex Eigenvalue Analysis) down to the point of complex eigenvalue analysis. The existing module PLOT was modified to accommodate plotting of the aerodynamic geometry. Both undeformed and deformed plots are available. Changes were made to the XYTRAN and XYPLOT modules for the purpose of making  $g - V$  and  $f - V$  plots. An upper Hessenberg method of complex eigenvalue extraction was added to module CEAD since this procedure is better suited to the requirements of flutter analysis than the two methods already available.

The completely new modules are the Aerodynamic Pool Distributor (APD), Geometry Interpolation (GI), Aerodynamic Matrix Generator (AMG), Aerodynamic Matrix Processor (AMP), Flutter Analysis Phase 1 (FA1), and Flutter Analysis Phase 2 (FA2). Module APD forms tables of aerodynamic data, defines the boundaries of the aerodynamic elements, and locates and orients displacement components at aerodynamic grid, or control, points. Module AMG evaluates the aerodynamic influence coefficient matrix at specified values of Mach number and reduced frequency, and determines the transformations needed to convert these matrices from the points required by the doublet lattice theory (one-quarter and three-quarter box chord stations) to the center of the aerodynamic boxes. Module GI generates the transformations required to give the structural displacements at the center of the aerodynamic boxes in terms of the deformations at the structural grid points. The AMP module calculates the generalized aerodynamic force matrices by using the mode shapes determined in the structural part of the rigid format (READ module), the aerodynamic matrices determined in AMG, and the transformation information calculated in GI. The module FA1 prepares the modal matrices for complex eigenvalue extraction by module CEAD. Also, the interpolation of the aerodynamic forces is carried out in this module, if a solution is required for a combination of parameters for which the generalized aerodynamic matrices were not determined previously. The module FA2 gathers data for reduction and presentation. For example, the velocity and frequency are determined from the eigenvalues calculated by CEAD, and a line of printer output is prepared for each loop through the flutter solution. The three modules FA1, CEAD, and FA2 are in a loop within the rigid format. This loop is repeated until solutions have been obtained for all the reduced frequencies, Mach numbers, and densities requested.

## PRELIMINARY APPLICATIONS

The NASTRAN flutter analysis has been applied to some simple geometric configurations. The results of three of these applications, a beamlike wing, a platelike wing, and a platelike wing with a folded tip, are presented in this section. The NASTRAN results are compared with other available analytical results and experimental data. Some discussion of the features of the NASTRAN analysis is included with the discussion of the applications.

The first application is the  $15^\circ$  swept wing shown in figure 4. Additional information concerning this wing may be found in references 8 and 9. This model was essentially a swept beam, and the NASTRAN structural model used consisted of 10 BAR elements as shown in the figure. The aerodynamic model consisted of 24 boxes arranged in six spanwise divisions of four chordwise boxes each. Unlike the requirements of the structural part of NASTRAN where the coordinates of each structural grid point are required input, a large number of aerodynamic boxes (and aerodynamic grid points which are located at the center of each box) are generated from a minimum amount of information. The aerodynamic boxes are assembled into panels, or groups, where each panel contains several boxes. For the beam example, all of the boxes belonged to a single panel. Only a single bulk data card (actually a parent card plus one continuation card) was required to define the aerodynamic boxes for this example. For each group, the only information required is the coordinates of the inboard and outboard leading-edge corners of the panel (points marked "a" and "b" in the fig.), the inboard and outboard chords (indicated by  $C_1$  and  $C_2$  in the fig.), the number of chordwise boxes, and the number of spanwise boxes if the boxes are to be equally spaced. If the boxes are not to be equally spaced, then the desired spacing is provided in terms of fraction chord and span divisions. The boxes for this example are equally spaced. Also, note that different coordinate systems were used to define the structural and aerodynamic models. A one-dimensional spline function was used for interpolation in this example. Presented in figure 5 are the results of the NASTRAN calculations for this wing at a Mach number of 0.45 and a density of  $1.185 \text{ kg/m}^3$ . Three modes were used in the analysis. The results are presented in the form of a  $g - V$  plot where only the critical root is shown. The circle symbols indicate the calculated points. The calculated flutter speed is determined by the point at which the line faired through the symbols crosses the  $g = 0$  axis. Indicated on the figure, in addition to the NASTRAN result, are the experimental flutter result from reference 8 and the calculated flutter result from reference 9 which were obtained using linearized lifting-surface theory. The NASTRAN calculated velocity is in good agreement with the experimental value. The calculated flutter speed from reference 9 is about 5 percent lower than the NASTRAN calculated value. The agreement with respect to flutter frequency is not so good.

The wing geometry, structural model, and aerodynamic model for the plate-like wing are presented in figure 6. Copies of NASTRAN computer-generated plots of the structural and aerodynamic models are presented in figure 7. The structural model consisted of 36 quadrilateral plate elements (QUAD2); the aerodynamic model consisted of 50 boxes, 10 spanwise divisions of unequal spacing, and five equally spaced chordwise boxes. As was done for the beam

model, the entire wing made up a single aerodynamic panel. The surface spline was used to perform the required structural-aerodynamic interface for this example. The calculated  $g - V$  curve is presented in figure 8, and only the critical locus of points is shown. These results are for a Mach number of 0.80 and a density of 1.200 kg/cu m. Four modes were used in the analysis. The solid symbols indicate calculated values for which the generalized aerodynamic forces were calculated. The open symbols indicate results obtained by using interpolated generalized aerodynamic forces. The calculated and interpolated results appear to lie on the same curve and could not be distinguished from one another had not different symbols been used. Tabulated on the figure are the NASTRAN calculated flutter speed and frequency, and some unpublished analytical results. Also included in the table are some NASTRAN calculated results for an aerodynamic model that had eight equally spaced chordwise boxes and the same spanwise arrangement shown in figure 6 for the 50-box case. The unpublished analytical results were obtained by using a doublet lattice computer program similar, but not identical, to the one modified for NASTRAN use. The surface spline was also used for the structural-aerodynamic interface in obtaining the unpublished result. The experimentally determined model natural frequencies were used to determine the generalized stiffnesses used in the unpublished results. Since the measured frequencies did not agree precisely with the calculated frequencies, some of the 7-percent difference between the two results may be attributed to this frequency difference. However, the results are still in good agreement. The two NASTRAN calculations gave essentially the same results.

The final application to be discussed is a platelike wing with a folded tip. A photograph of this model is presented in figure 9, and the geometry, structural model, and aerodynamic model are presented in figure 10. The tip fin is inclined with respect to the wing by  $60^\circ$ . Copies of NASTRAN generated computer plots of the structural and aerodynamic elements are presented in figure 11. The wing portion of this model was the same as the platelike wing previously discussed, and this portion was modeled in the same fashion as the plate wing (36 QUAD2 structural elements and 50 aerodynamic boxes comprising one aerodynamic panel). An additional 60 QUAD2 structural elements were used to model the folded tip. The folded tip was a separate aerodynamic panel and was composed of a total of 50 boxes that were arranged into five equal chordwise divisions and 10 unequal spanwise divisions as indicated in the figure. One provision provided by the program is that there may or may not be aerodynamic interference, or coupling, between boxes located in different panels, or groups, depending on the user to make the selection. This feature allows for the omission of coupling when it is known to be unimportant and thereby reduces the time required to compute the aerodynamic matrices, or allows for the independent investigation of aerodynamic interference effects. In the present example, aerodynamic coupling between the wing panel and the tip panel was included. The surface spline option was used to perform the required aerodynamic-structural interface. Four different spline functions were used, two for each aerodynamic panel. The interpolation for the 25 inboard wing aerodynamic boxes used one spline function, and the 25 outboard boxes used another spline function. The same type of arrangement was used for the tip fin. Since the analyst specifies the structural grid points that are to be used for interpolating for each aerodynamic box, it is not necessary that a single spline function be used for each aerodynamic panel.

The results of NASTRAN calculations for a Mach number of 0.90 and a density of  $0.861 \text{ kg/m}^3$  are presented in figure 12 in the form of the  $g - V$  plot for the critical eigenvalue. Four modes were used in this analysis. The data obtained by using calculated generalized aerodynamic forces are indicated by the solid symbols in the figure, and the results using interpolated generalized aerodynamic forces are indicated by the open symbols. The comparison of the results using calculated generalized aerodynamic forces with those obtained using interpolated forces indicates that they all lie on the same  $g - V$  curve. Also tabulated on the figure are an unpublished calculated result and an unpublished wind-tunnel experimental result. The unpublished calculated result was obtained in a fashion similar to that previously described for the platelike wing example. The two calculated results are in good agreement with respect to both flutter velocity and frequency. The experimental flutter velocity is about 9 percent lower than the NASTRAN calculated value. Both calculated flutter frequencies are somewhat higher than the experimental frequency.

In discussing these three applications, some mention has been made of the simplicity of the input data requirements associated with the aerodynamics portion of the NASTRAN program. This point is somewhat dramatically indicated by the fact that for the wing with tip fin case, of a total of 401 bulk data cards used, only 28 were directly associated with the aerodynamics or flutter solution.

Since the NASTRAN flutter analysis is relatively new, its efficiency has not been fully evaluated nor have all of its potential options been exercised. However, it is of interest to examine some of the central processing unit (CPU) computer times required by some of the individual functional modules for a program execution. Presented in figure 13 is a listing of CPU times for the CDC 6600 computer obtained for the wing with the folded tip fin. In this case, five modes were calculated by the real eigenvalue module, and the four lowest modes were used in the flutter analysis. The generalized aerodynamic forces were determined at three values of reduced frequency and interpolated to two additional values so the flutter eigenvalue problem was solved five times. Additional information describing this example is shown on the figure. Also included on the figure are the total CPU time, the peripheral processor time (CPU), and calls to the operating system (O/S calls).

#### CONCLUDING REMARKS

A subsonic flutter analysis capability has been developed for NASTRAN. This flutter analysis is of the modal type, uses doublet lattice unsteady aerodynamic forces, and solves the flutter equations by using the k-method. One-dimensional and surface spline functions are used to transform from aerodynamic degrees of freedom to structural degrees of freedom. This capability has been incorporated into a version of NASTRAN, and this version has been installed on the CDC 6000 series computers at the Langley Research Center. This version is in a developmental stage and is not now available for general release. In this paper, a general description of the new flutter analysis rigid format has been presented. Results of some preliminary applications of

the NASTRAN flutter analysis to a beamlike wing, a platelike wing, and a plate-like wing with a folded tip have been presented, and these results compared with existing experimental and analytical results.

#### REFERENCES

1. Anon.: NASTRAN: Users' Experiences. NASA TM X-2378, 1971.
2. Harder, Robert L., MacNeal, Richard H., and Doggett, Robert V., Jr.: A Design Study for the Incorporation of Aeroelastic Capability Into NASTRAN. NASTRAN: Users' Experiences, NASA TM X-2378, Sept. 1971, pp. 779-795.
3. Albano, Edward, and Rodden, William P.: A Doublet Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flows. AIAA Journal, Vol. 7, No. 2, Feb. 1969, pp. 279-285.
4. Giesing, J. P., Kalman, T. P., and Rodden, W. P.: Subsonic Unsteady Aerodynamics for General Configurations: Part II, Volume I, Application of the Doublet-Lattice Method and the Method of Images to Lifting-Surface/Body Interference. AFFDL-TR-71-5, Part II, Vol. I, Aug. 1971, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio.
5. Giesing, J. P., Kalman, T. P., and Rodden, W. P.: Subsonic Unsteady Aerodynamics for General Configurations: Part II, Volume II, Computer Program N5KA. AFFDL-TR-71-5, Part II, Vol. II, April 1972, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio.
6. Greville, T. N. E., ed.: Theory and Application of Spline Functions. Academic Press, 1969.
7. Harder, Robert L., and Desmarais, Robert N.: Interpolation Using Surface Splines. AIAA Journal of Aircraft, Vol. 9, No. 2, Feb. 1972, pp. 189-191.
8. Tuovila, W. J., and McCarty, John Locke: Experimental Flutter Results for Cantilever-Wing Models at Mach Numbers up to 3.0. NACA RM L55E11, 1955.
9. Yates, E. Carson, Jr., and Bennett, Robert M.: Use of Aerodynamic Parameters From Nonlinear Theory in Modified-Strip-Analysis Flutter Calculations for Finite-Span Wings at Supersonic Speeds. NASA TN D-1824, 1963.

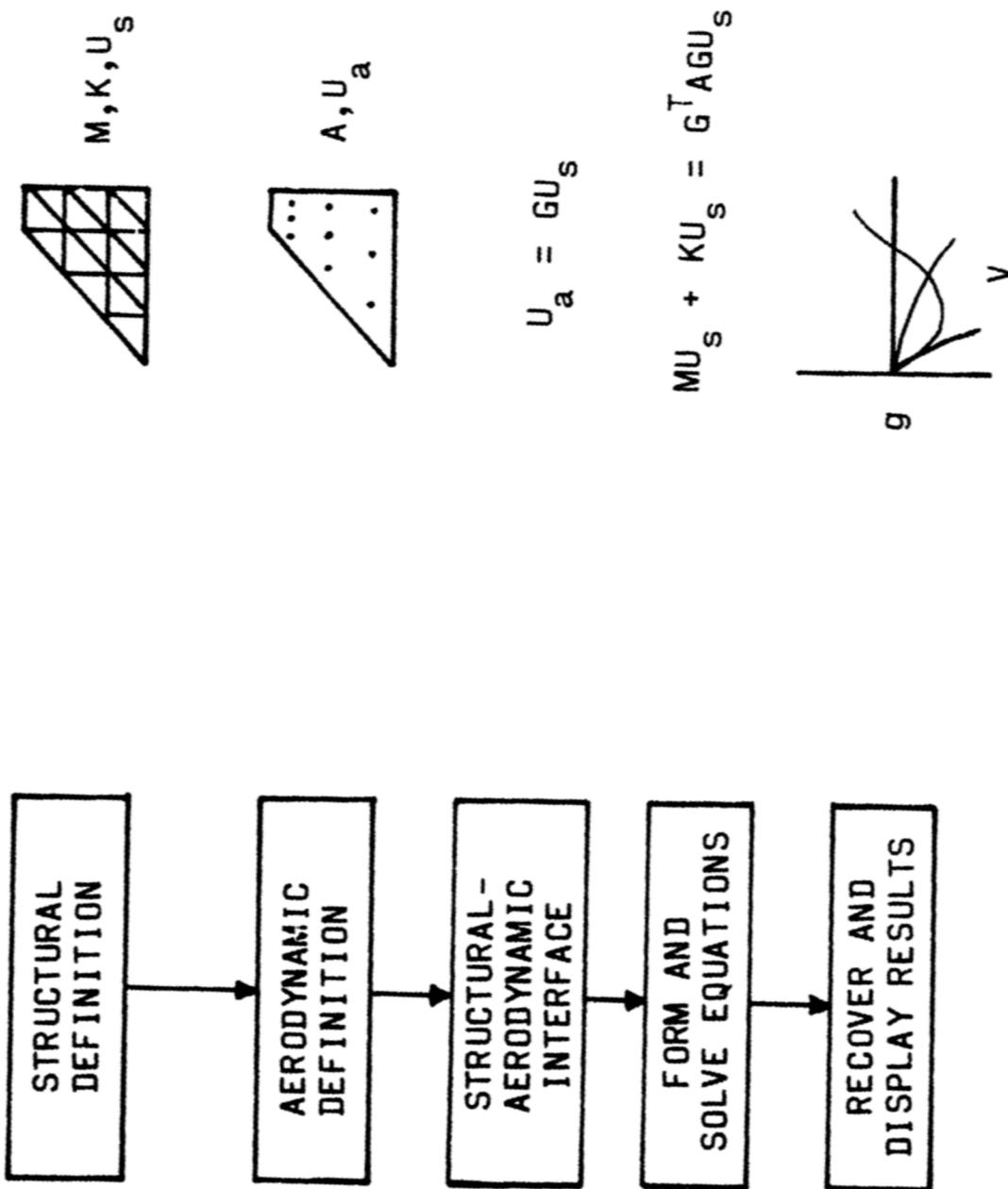


Figure 1.- Steps required in flutter analysis.

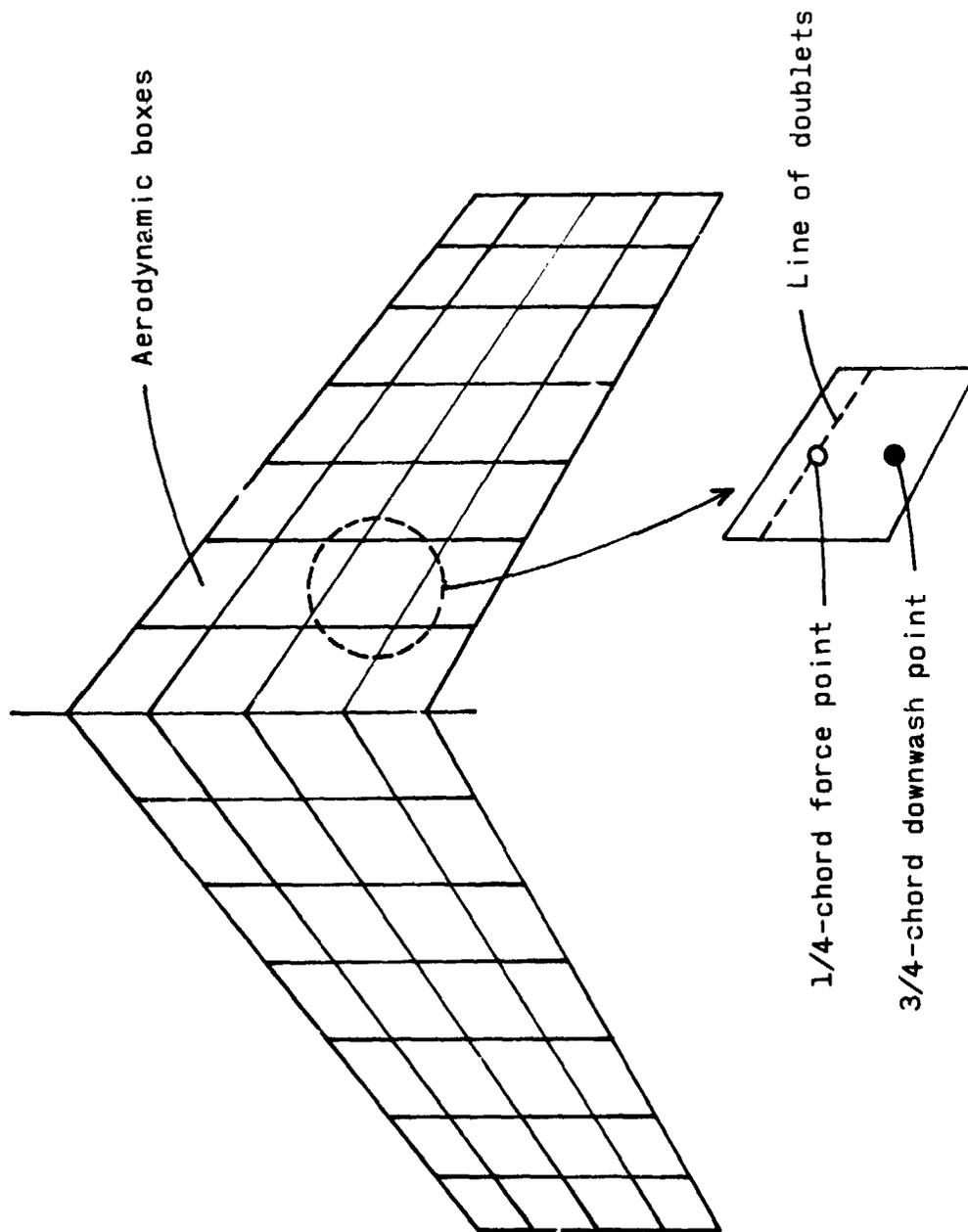


Figure 2.— Sample arrangement of doublet lattice aerodynamic boxes.

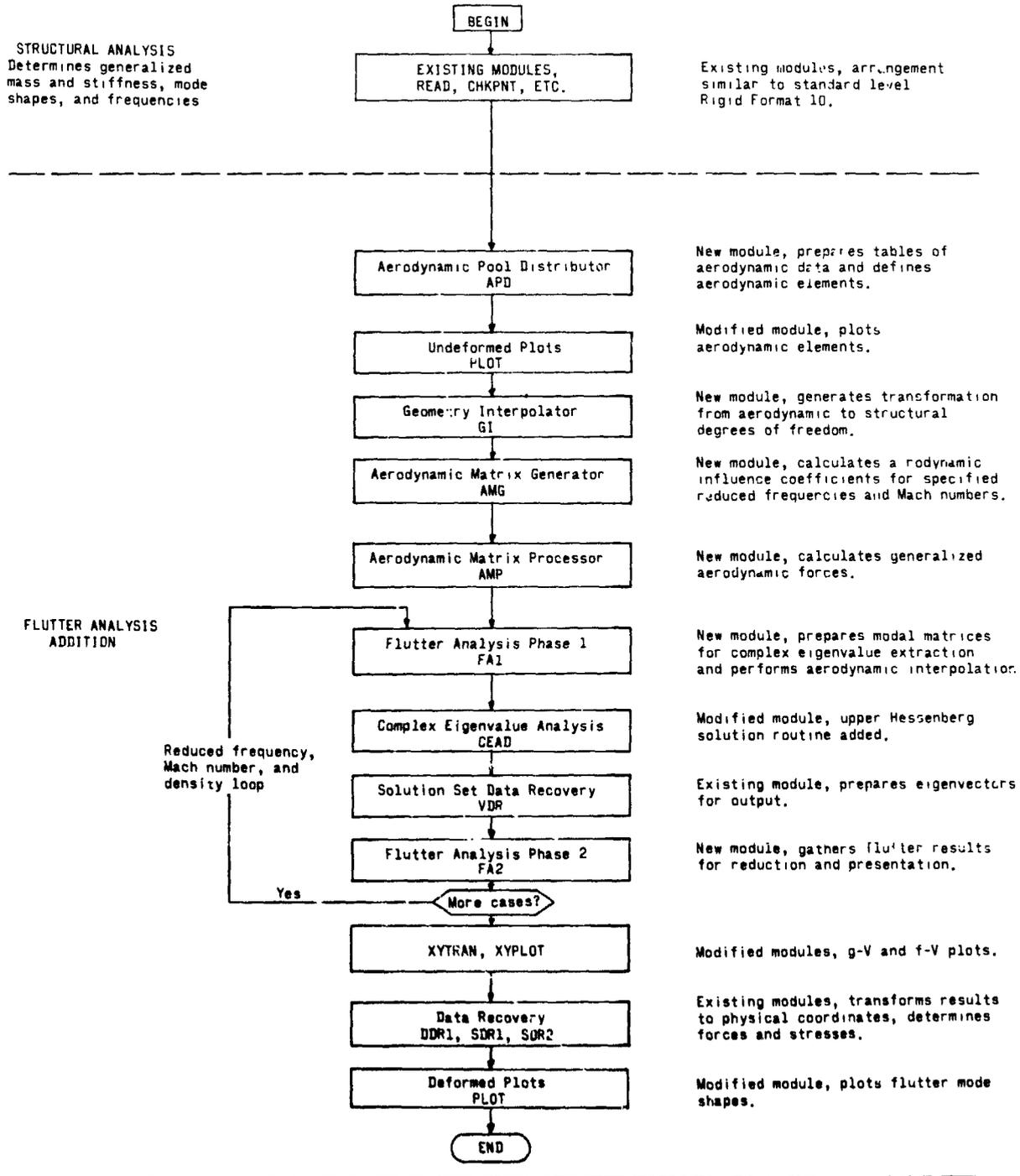


Figure 3.- Block diagram of flutter analysis rigid format.

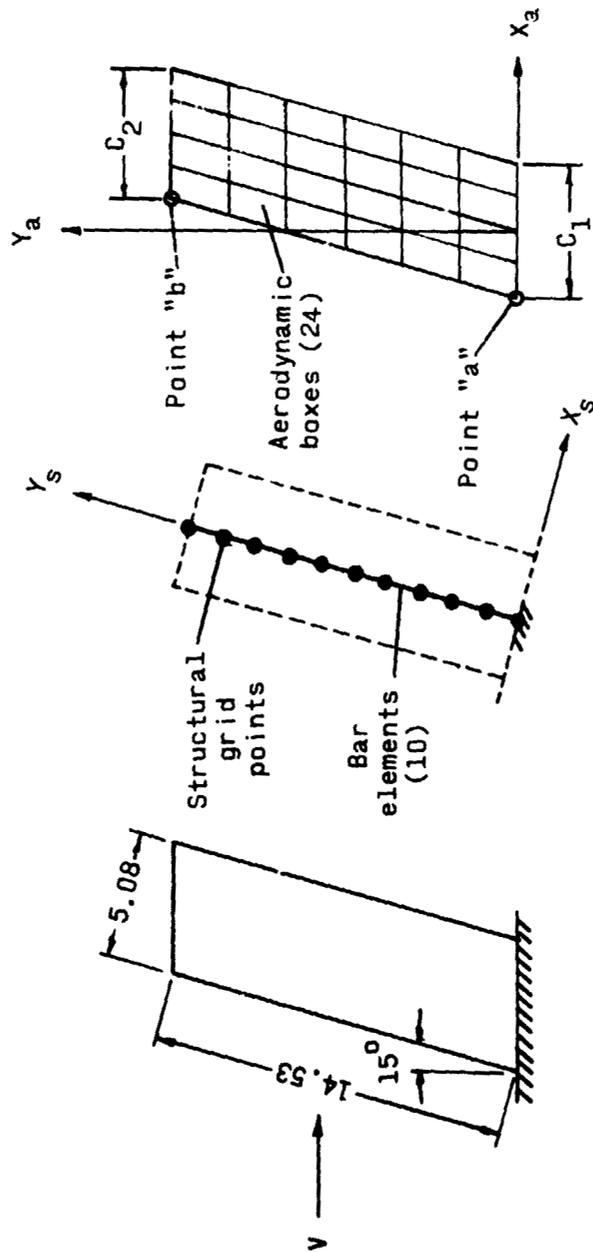


Figure 4.— Geometry, structural modeling, and aerodynamic modeling of beamlike wing. (All linear dimensions are in centimeters.)

METHOD	$V_f$ , m/sec	$f_f$ , Hz
—○— NASTRAN	152.9	140
□ Calc., Ref. 9	145.3	178
△ Exp., Ref 8	150.9	120

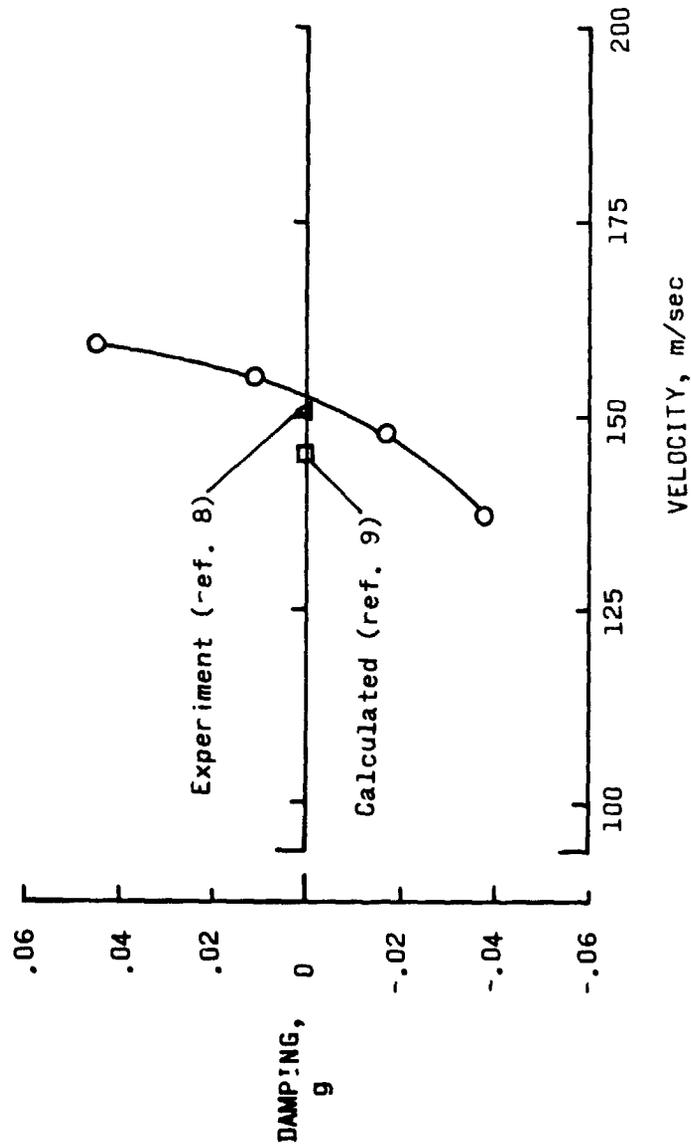
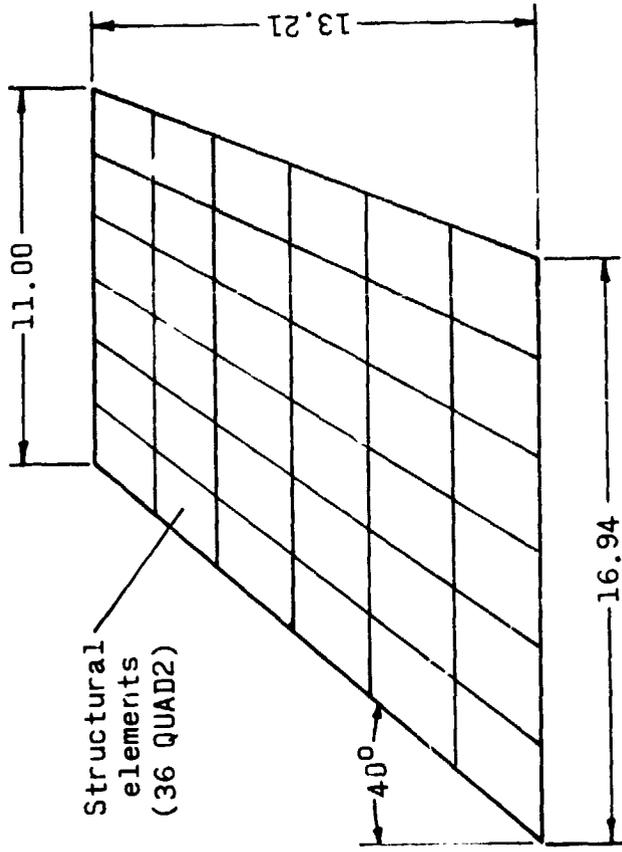
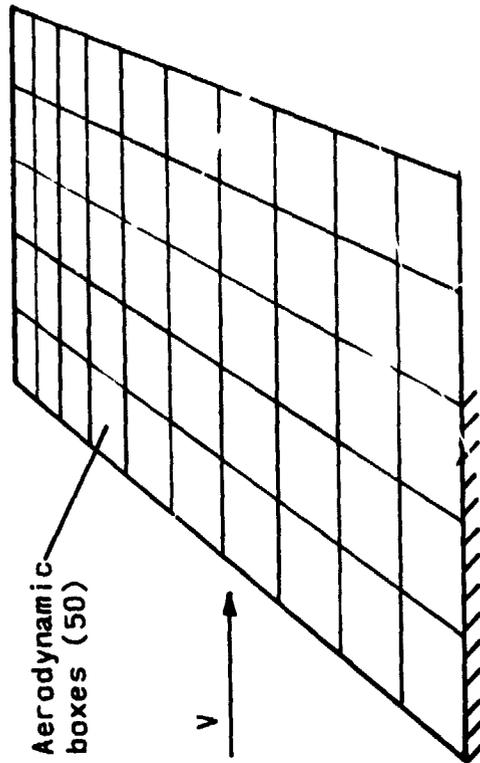


Figure 5.— Beamlike wing flutter results for a Mach number of 0.45 and a density of 1.185 kg/m<sup>3</sup>.



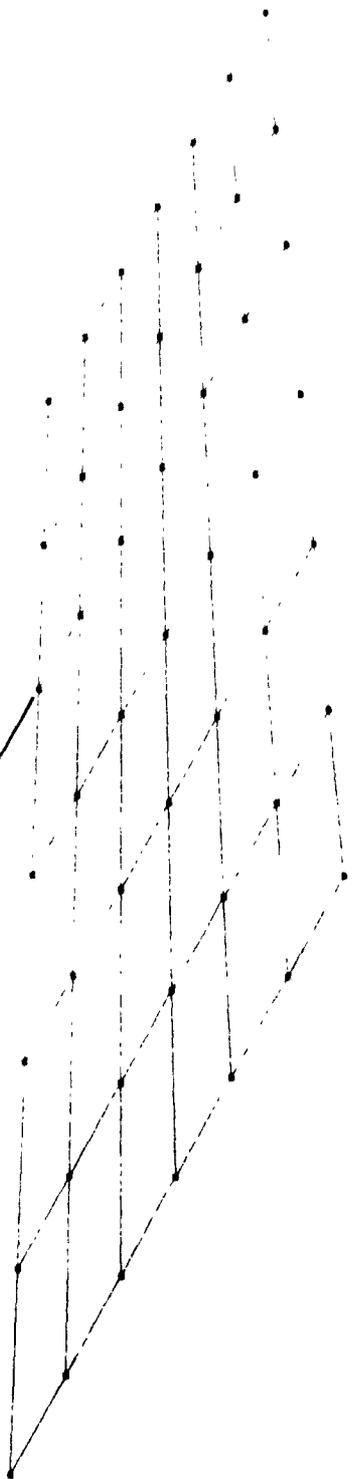
GEOMETRY AND STRUCTURAL MODEL



AERODYNAMIC MODEL

Figure 6.— Geometry, structural modeling, and aerodynamic modeling of plate-like wing. (All linear dimensions are in centimeters.)

Structural  
grid point



STRUCTURAL MODEL

Aerodynamic  
grid point



AERODYNAMIC MODEL

Figure 7.- Computer-generated plots of structural and aerodynamic models for flatlike wing.

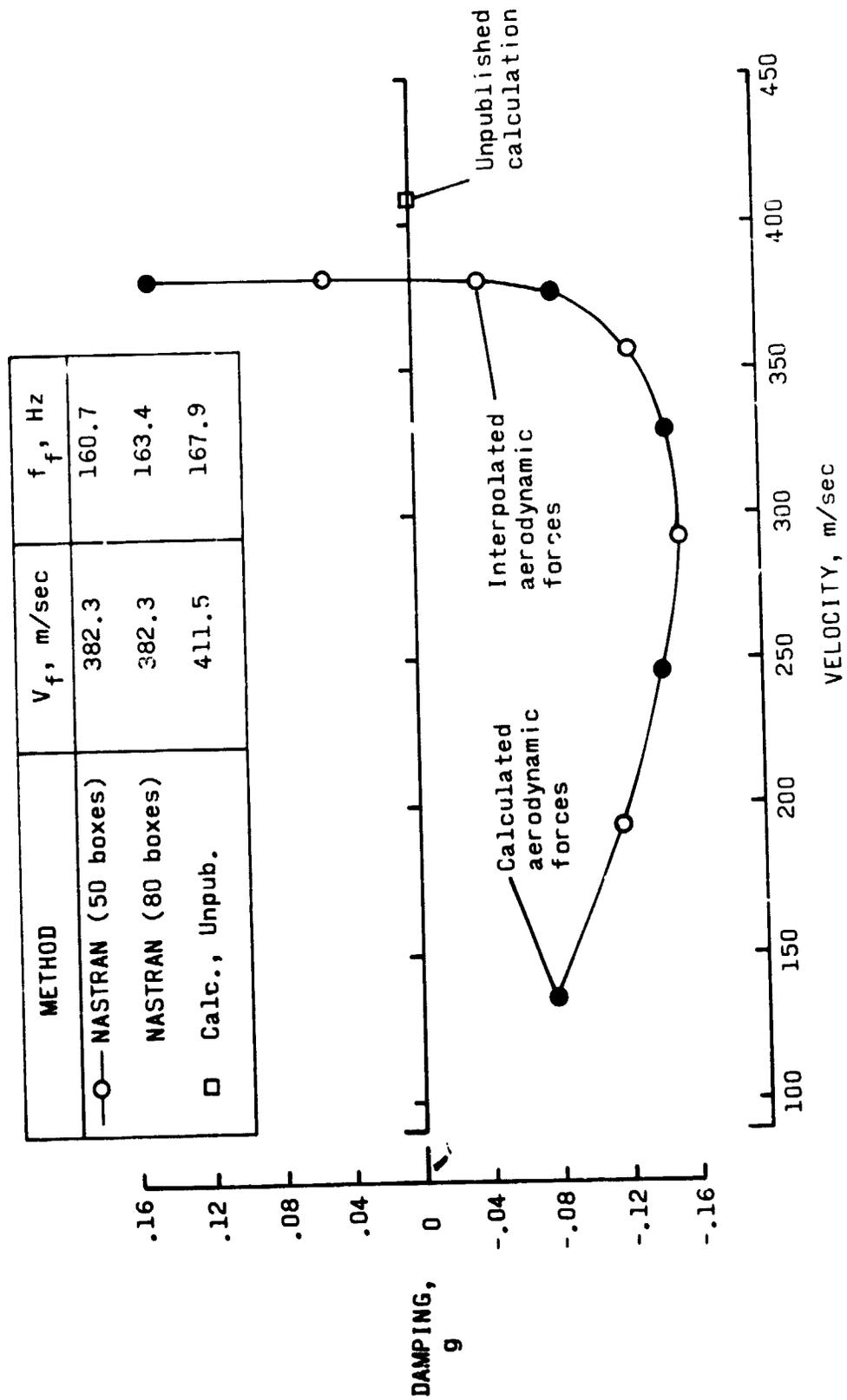


Figure 8.—Platelike wing flutter results for a Mach number of 0.80 and a density of  $2.700 \text{ kg/m}^3$ .



Figure 9. - Photograph of platelike wing with folded tip fin.

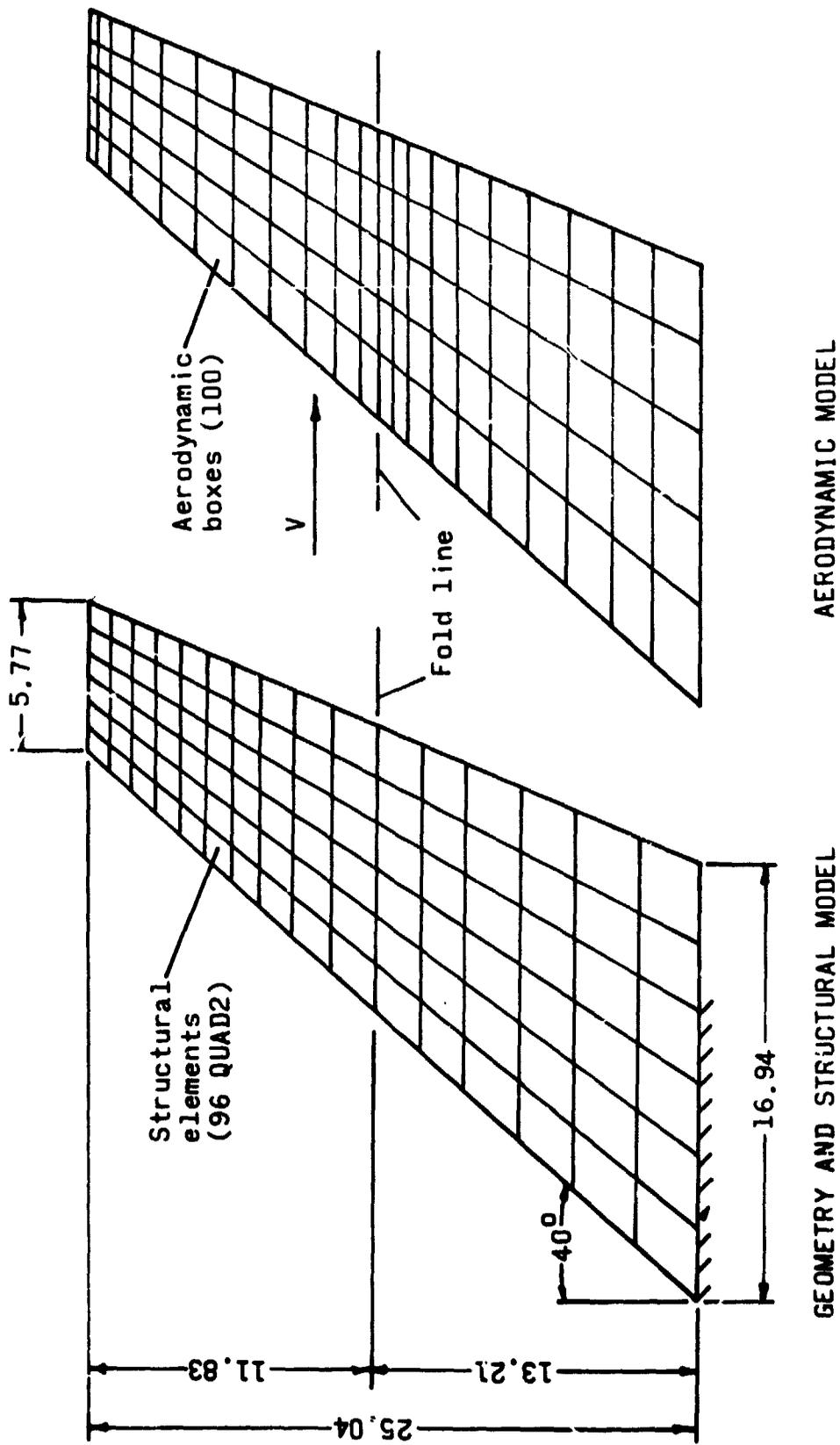
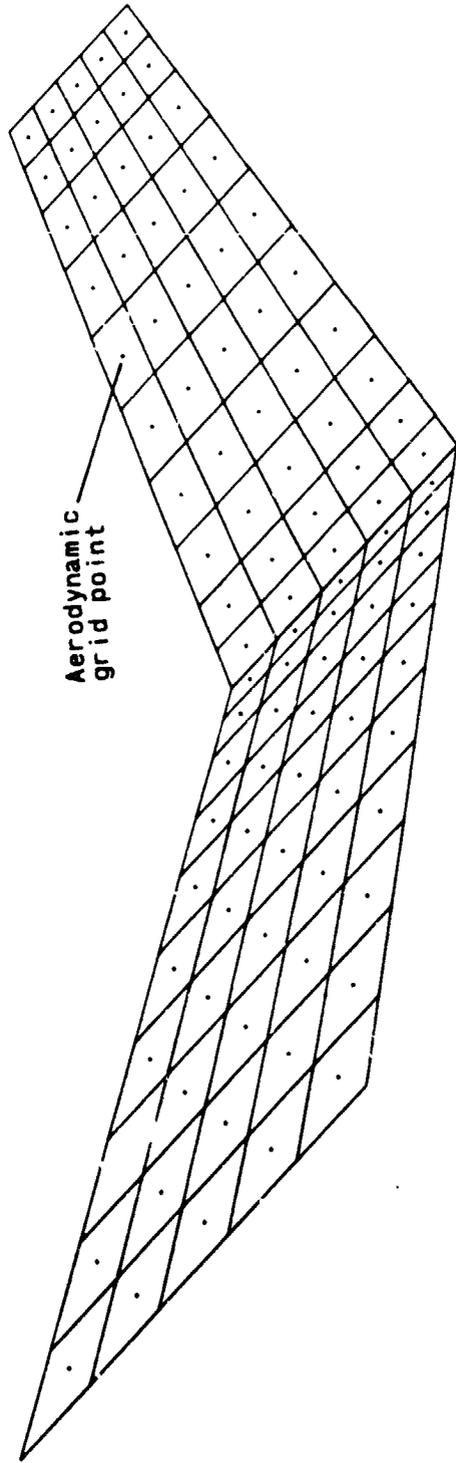
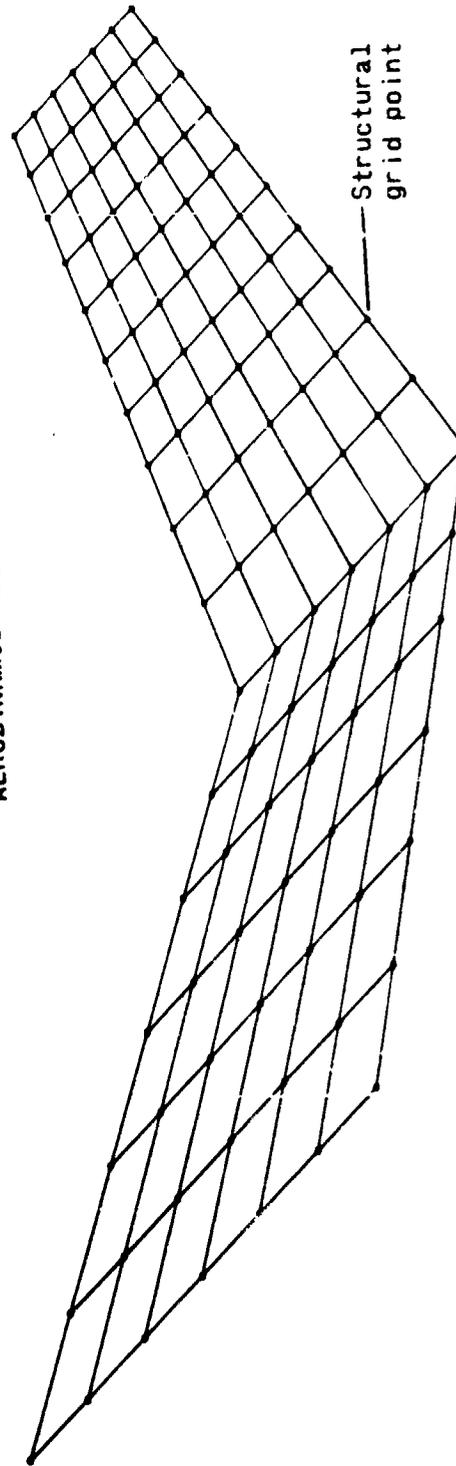


Figure 10. - Geometry, structural model, and aerodynamic model of platelike wing with folded tip fin. (All linear dimensions are in centimeters.)



AERODYNAMIC MODEL



STRUCTURAL MODEL

Figure 11.-- Computer-generated plots of structural and aerodynamic models for wing with folded tip fin.

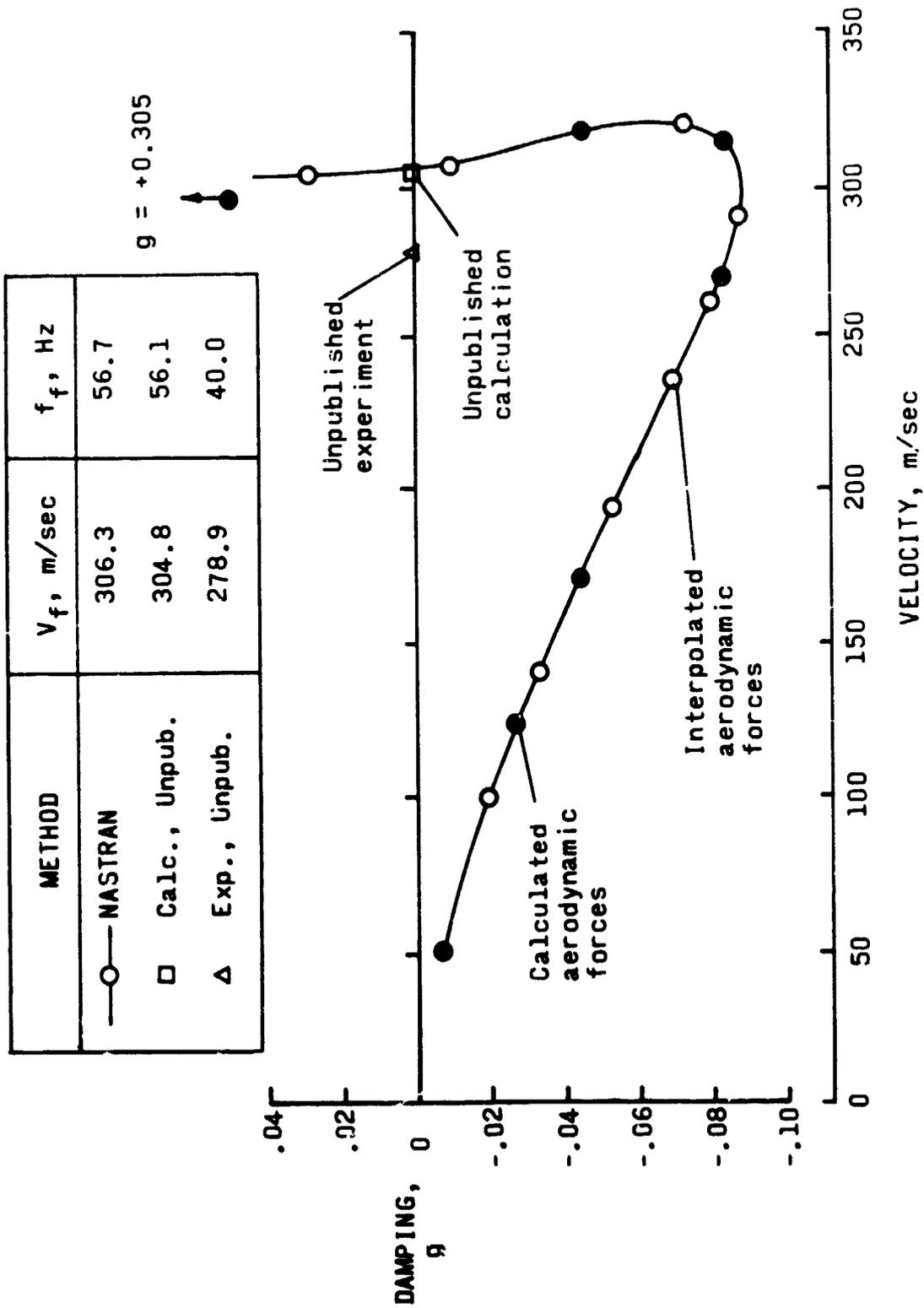


Figure 12.— Platelike wing with folded tip fin flutter results for a Mach number of 0.90 and a density of  $0.861 \text{ kg/m}^3$ .

NASTRAN MODULE	CPU TIME, sec	FUNCTION
IFP	2.958	Sort input data, set up restart tables, etc.
GPI to READ	191.208	Form structural matrices (96 QUAD2 elements)
READ thru GKAM	396.798	Real eigenvalue analysis (5 modes)
APD	.962	Generate aerodynamic elements (100 boxes)
GI	25.196	Geometry interpolation (4 surface splines)
AMG	195.404	Generate aerodynamic influence coefficient matrices (3 values of reduced frequency)
AMP	126.566	Generate generalized aerodynamic matrices (4 modes)
FA1-CEAD-FA2	8.648	Flutter solution for 5 reduced frequencies, aerodynamic interpolation for 2 reduced frequencies
XYTRAN, XY?PLOT	.510	g-V plot and f-V plot

Total CPU time = 966.3 sec  
Total PPU time = 2822.7 sec  
O/S calls = 36401

Figure 13.- Sample flutter solution computer times.